

# On Types, Intension and Compositionality

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**Abstract.** In this paper we demonstrate that a number of challenging problems in the semantics of natural language, namely the treatment of the so-called intensional verbs and the semantics of nominal compounds, can be adequately resolved in the framework of compositional semantics, if a strongly-typed ontological structure is assumed. In addition to suggesting a proper treatment of nominal compounds and intensional verbs within the framework of compositional semantics, we briefly discuss the nature of this ontological type system and how it may be constructed.

## 1 The Semantics of Nominal Compounds

The semantics of nominal compounds have received considerable attention by a number of authors, most notably (Kamp & Partee, 1995; Fodor & Lepore, 1996; Pustejovsky, 2001), and to our knowledge, the question of what is an appropriate semantics for nominal compounds has not yet been settled. In fact, it seems that the problem of nominal compounds has presented a major challenge to the general program of compositional semantics in the Montague (1973) tradition, where the meaning of a compound nominal such as  $[N_1 N_2]$  is generally given as follows:

$$(1) \quad \|N_1 N_2\| = F(\|N_1\|, \|N_2\|)$$

In the simplest of cases, the compositional function  $F$  is usually taken to be a conjunction (or intersection) of predicates (or sets). For example, assuming that  $\text{red}(x)$  and  $\text{apple}(x)$  represent the meanings of *red* and *apple*, respectively, then the meaning of a nominal such as *red apple* is usually given as

$$(2) \quad \|\text{red apple}\| = \{x \mid \text{red}(x) \wedge \text{apple}(x)\}$$

What (2) says is that something is a *red apple* if it is *red* and *apple*. This simplistic model, while seems adequate in this case (and indeed in many other instances of

similar ontological nature), clearly fails in the following cases, all of which involve an adjective and a noun:

- (3) *former senator*
- (4) *fake gun*
- (5) *alleged thief*

Clearly, the simple conjunctive model, while seems to be adequate for situations similar to those in (2), fails here, as it cannot be accepted that something is *former senator* if it is *former* and *senator*, and similarly for (4) and (5). Thus, while conjunction is one possible function that can be used to attain a compositional meaning, there are in general more complex functions that might be needed for other types of ontological categories. In particular, what we seem to have is something like the following:

- (6)  $\| \text{red apple} \| = \{x \mid x \text{ is red and } x \text{ is apple}\}$
- (7)  $\| \text{former senator} \| = \{x \mid x \text{ was but is not now a senator}\}$
- (8)  $\| \text{fake gun} \| = \{x \mid x \text{ looks like but is not actually a gun}\}$
- (9)  $\| \text{alleged thief} \| = \{x \mid x \text{ could possibly turn out to be a thief}\}$

It would seem, then, that different ontological categories require different compositional functions to compute the meaning of the whole from the meanings of the parts. In fact, the meaning (intension) of some compound might not be captured without resorting to temporal and/or modal operators. This has generally been taken as an argument against compositionality, in that there does not seem to be an answer as to what the compositional semantic function  $F$  in  $\|N_1 N_2\| = F(\|N_1\|, \|N_2\|)$  might be. We believe, however, that this is a fallacious argument in that the problem is not due to compositionality but in 'discovering' a number of semantic functions that could account for all nominal compounds of different ontological categories. Moreover, we believe that the answer lies in assuming a richer type structure than the flat type system typically assumed in Montague-style semantics.

## 2 Ontology and the Semantics of Adjectives

In (2) we stated that the meaning of some adjectives. The question however is what "kinds" of adjectives are specifically intersective. It would seem that for constructions of the form  $[A N]$  where  $A$  is a physical property (such as *red*, *large*, *heavy*, etc.) and  $N$  is a object of type *PhysicalThing* (such as *car*, *person*, *desk*, etc.), the meaning of  $[A N]$  can be obtained as follows:

$$(10) \quad \|A N\| = \{x \mid A_{\text{PhysicalProperty}}(x) \wedge N_{\text{PhysicalThing}}(x)\}$$

Note here that the above expression is not a statement about the meaning of any particular adjective. Instead, what (10) simply states is that some adjectives, such as *large*, *heavy*, etc. are intersective. Thus, in  $\| \text{large table} \| = \{x \mid \text{large}(x) \wedge \text{table}(x)\}$ .



for example, it is assumed that the meaning of *large*, namely the predicate  $\text{large}(x)$  has been defined. Although the semantics of such adjectives is not our immediate concern here, it must be pointed out that semantics of such (intersective) adjectives, which are presumably the simplest, can be quite involved, as these adjectives are very context-sensitive – clearly the sense of 'large' in 'large elephant' is quite different from the sense of 'large' in 'large bird'. Assuming a predicate  $\text{typical}_{a,T}(x)$ , which is true of some object  $x$  of type  $T$  if  $x$  is a typical object with respect to one of its attributes  $a$  is defined, then the meanings of such adjectives as *large* and *heavy*, for example, could be defined as follows, where  $x :: T$  refers to an object  $x$  of type  $T$ :

$$(11) \quad \text{large} \Rightarrow (\forall x :: \text{PhysicalThing})(\text{large}(x) \equiv_{df} \lambda P [P(x) \wedge (\exists y :: \text{PhysicalThing})(P(y) \wedge \text{typical}_{size}(y) \wedge \text{size}(x, s_1) \wedge \text{size}(y, s_2) \wedge (s_1 >> s_2))]))$$

$$(12) \quad \text{heavy} \Rightarrow (\forall x :: \text{PhysicalThing})(\text{heavy}(x) \equiv_{df} \lambda P [P(x) \wedge (\exists y :: \text{PhysicalThing})(P(y) \wedge \text{typical}_{weight}(y) \wedge \text{weight}(x, w_1) \wedge \text{weight}(y, w_2) \wedge (w_1 >> w_2))]))$$

What (1) and (2) say is the following: that some  $P$  object  $x$  is a *large* (*heavy*)  $P$ , iff it has a *size* (*weight*) which is larger than the *size* (*weight*) of another  $P$  object,  $y$ , which has a typical *size* (*weight*) as far as  $P$  objects go. It would seem, then, that the meaning of such adjectives is tightly related to some attribute (*large/size*, *heavy/weight*, etc.) of the corresponding concept. Thus, such adjectives, while they are intersective, are context-dependent: their meaning is fully specified only in the context of a specific concept.

One of the main points that we like to make in this paper is that, like intersective adjectives, non-intersective adjectives also have a compositional meaning, although the compositional function might be more involved than simple conjunction. For example, we argue that the following are reasonable definitions for *fake*, *former* and *alleged*:

$$(13) \quad (\forall x :: \text{PhysicalArtifact})(\text{fake}(x) \equiv_{df} \lambda P [(\exists y :: \text{Physical})(\neg P(x) \wedge P(y) \wedge \text{similar}_{\{shape, size\}}(x, y))])$$

$$(14) \quad (\forall x :: \text{Role})(\text{former}(x) \equiv_{df} \lambda P [(\exists t)((t < \text{now}) \wedge P(x, t) \wedge \neg P(x, \text{now}))])$$

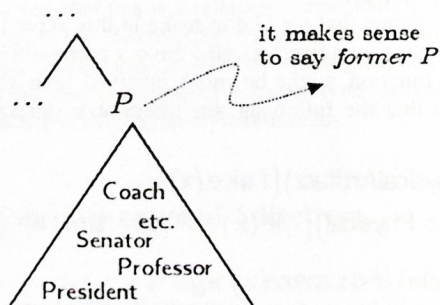
$$(15) \quad (\forall x :: \text{Role})(\text{alleged}(x) \equiv_{df} \lambda P [(\exists t)((t > \text{now}) \wedge \neg P(x, \text{now}) \wedge \Diamond P(x, t))])$$

That is, 'fake' applies to some concept  $P$  as follows: a certain physical object  $x$  is a *fake*  $P$  iff it is not a  $P$ , but looks like (in certain respects) to something else, say  $y$ , which is actually a  $P$ . On the other hand, what (14) says is the following: a

certain  $x$  is a former  $P$  iff  $x$  was a  $P$  at some point in time in the past and is not now a  $P$ . Finally, what (15) says is that something is an 'alleged'  $P$  iff it is not now known to be a  $P$ , but could possibly turn out to be a  $P$  at some point in the future.

It is interesting to note here that the intension of *fake* and that of *former* and *alleged* was in one case represented by recourse to possible worlds semantics (the case of (14) and (15)), while in (13) the intension uses something like structured semantics, assuming that similar <sub>$\{A_1, A_2, \dots, A_n\}$</sub> ( $x, y$ ) which is true of some  $x$  and some  $y$  if  $x$  and  $y$  share a number of important features, is defined. What is interesting in this is that it suggests that possible-worlds semantics and structured semantics are not two distinct alternatives to representing intensionality, as has been suggested in the literature, but that in fact they should co-exist.

Additionally, several points should also be made here. First, the representation of the meaning of *fake* given in (13) suggests that *fake* expects a concept which is of type **PhysicalArtifact**, and thus something like *fake idea*, or *fake song*, for example, should sound meaningless, from the standpoint of commonsense<sup>1</sup>. Second, the representation of the meaning of *former* given in (14) suggests that *former* expects a concept which has a time dimension, i.e. is a temporal concept. Finally, we should note here that our ultimate goal of this type of analysis is to discover the ontological categories that share the same behavior. For example, an analysis of the meaning of *former*, given in (14), suggests that there are a number of ontological categories that seem to share the same behavior, and could thus replace  $P$  in (14), as implied by the fragment hierarchy below.



### 3 Types, Predicates and Logical Concepts

In "Logic and Ontology" Cocchirella (2001) argues for a view of logic as a language in contrast with the view of logic as a language. In the latter, logic is

<sup>1</sup> One can of course say *fake smile*, but this is clearly another sense of *fake*. While *fake gun* refers to a gun (which is of type **Artifact**) that is not real, *fake smile* refers to a dishonest smile, or a smile that is not genuine.



viewed as an "abstract calculus that has no content of its own, and which depends on set theory as a background framework by which such a calculus might be syntactically described and semantically interpreted." In the view of *logic* as a *language*, however, logic has content, and "ontological content in particular." This view however necessitates the use of type theory, as opposed to set theory as the background framework. It is this view that we advocate here, and in our opinion, problems in the semantics of natural language cannot be resolved until a logic that is grounded in type theory and predication (as opposed to set membership) is properly formulated. In this section we discuss the building blocks of such a program.

### 3.1 Types vs. Predicates

In formal (programming) languages we write statements such as 'int x', which is a type declaration statement meaning that x is an object of type int. However, in programming languages we do not have procedures that verify (somehow) if some object is of a certain type - that is we do not have a predicate such as `int(x)` that takes some object x and returns 'true' if x is an int and 'false' otherwise. Clearly, the type and the corresponding predicate are related, and in particular, a predicate such as `int(x)` is true of some object x if has all the properties of the type int.

Like objects in formal (programming) languages, commonsense objects have a type, and a corresponding predicate that verifies if a certain object is of a specific type. For instance, our ontology has a type hierarchy that contains the following fragment:

(16) `Piano`  $\supset$  `Instrument`  $\supset$  ...  $\supset$  `Artifact`  $\supset$  ...  $\supset$  `PhysicalThing`  $\supset$  `Thing`

Corresponding to these types there are predicates such as `piano(x)`, `instrument(x)`, etc. Moreover, a predicate such as `piano(x)` is true of some object x just in case x happens to be a piano. That is, such concepts correspond to what Cocchiarrella (2001) refers to as 'first intentions', i.e., concepts abstracted directly from physical reality. The point here is that what makes some object x a piano, for example (or, what makes `piano(x)` true of some object x) is determined directly from physical reality. Such 'first intentions' concepts should be contrasted with concepts that are about 'second intentions', which, according to Cocchiarrella are "concepts abstracted wholly from the 'material' content of first intentions", using the logical apparatus. Thus first intention concepts are in some sense 'ontological concepts', while second intention concepts can be thought of as 'logical concepts'.

Continuing with our example, `piano(x)` would be an ontological concept, while `pianist(x)`, for example, is a concept that is logically defined using the concept `piano(x)`, and perhaps other 'first intention' concepts. In other words, what makes `pianist(x)` true of some x is not physical reality but some set of logical conditions. This can be stated as follows:

(17)  $(\forall x :: \text{Artifact})(\text{piano}(x) \equiv \text{NuralNetPatternRecogProc}(x))$

- (18)  $(\forall x :: \text{Human})(\text{pianist}(x) \equiv_{\text{df}} (\exists a :: \text{Activity})(\exists p :: \text{Artifact})(\text{playing}(a) \wedge \text{piano}(p) \wedge \text{agent}(a, x) \wedge \text{object}(a, \text{Music}) \wedge \text{instrument}(a, p)))$

That is, something is a piano if it looks like, sounds like, feels like, etc. what we call 'piano'. On the other hand, what (18) says is the following: any **Human**  $x$ , is a **pianist**, iff there is some playing activity and some **Artifact** which is a piano where the object of this playing activity is **Music** and the instrument of this activity is a piano.

### 3.2 Compound Nominals Revisited

The problem of compound nominals in the case of noun-noun combinations has traditionally been due to the various relations that are usually implicit between the nouns (see Weiskopf, forthcoming). For example, consider the following:

- (19)  $\|\text{brick house}\| = \{x \mid x \text{ is a house that is made of brick}\}$   
 (20)  $\|\text{dog house}\| = \{x \mid x \text{ is a house that is made for a dog}\}$   
 (21)  $\|\text{beer drinker}\| = \{x \mid x \text{ often drinks beer}\}$   
 (22)  $\|\text{beer factory}\| = \{x \mid x \text{ is a factory that makes beer}\}$

Thus, while a **brick house** is a house 'made of' bricks, a **dog house** is a house that is 'made for' a dog. It would seem, then, that the relation implicitly implied between the two nouns differ with different noun-noun combinations. However, assuming the existence of a strongly-typed ontology might result in identifying a handful of implicit relations that can account for all patterns. Consider for example the following:

- (23)  $\|\text{brick house}\| = \{x : \text{Artifact} \mid \text{house}(x) \wedge (\exists y : \text{Substance})(\text{brick}(y) \wedge \text{madeOf}(x, y))\}$   
 (24)  $\|\text{paper cup}\| = \{x : \text{Artifact} \mid \text{cup}(x) \wedge (\exists y : \text{Substance})(\text{paper}(y) \wedge \text{madeOf}(x, y))\}$   
 (25)  $\|\text{plastic knife}\| = \{x : \text{Artifact} \mid \text{knife}(x) \wedge (\exists y : \text{Substance})(\text{plastic}(y) \wedge \text{madeOf}(x, y))\}$

It would seem, therefore, that the same semantic relation, namely **madeOf**, is the relation that is implicit between all  $[N_1, N_2]$  combinations when  $N_1$  is an **Artifact** and  $N_2$  is a **Substance**. Similarly, it would seem that the same semantic relation underlies all  $[N_1, N_2]$  combinations when  $N_1$  is a **Human** and  $N_2$  is a **Substance**, where  $P$  should be read as 'it is often the case that  $P$ ', or 'generally,  $P$ '.



- (26)  $\llbracket \text{beer drinker} \rrbracket = \{x :: \text{Human} \mid (\exists y :: \text{Substance})(\text{beer}(y) \wedge \Delta(\text{drinks}(x,y)))\}$   
 (27)  $\llbracket \text{cigar smoker} \rrbracket = \{x :: \text{Human} \mid (\exists y :: \text{Substance})(\text{cigar}(y) \wedge \Delta(\text{smokes}(x,y)))\}$

#### 4 The So-Called Intensional Verbs

In (Montague, 1969) Montague discusses a puzzle pointed out to him by Quine which can be illustrated by the following examples:

- (16)  $\llbracket \text{John painted a unicorn} \rrbracket = (\exists x)(\text{unicorn}(x) \wedge \text{painted}(j,x))$   
 (17)  $\llbracket \text{John found a unicorn} \rrbracket = (\exists x)(\text{unicorn}(x) \wedge \text{found}(j,x))$

The puzzle Quine was referring to was the following: both translations admit the inference  $(\exists x)(\text{unicorn}(x))$  – that is, both sentences imply the existence of a unicorn, although it is quite clear that such an inference should not be admitted in the case of (17). According to Montague, the obvious difference between (16) and (17) must be reflected in an ontological difference between *find* and *paint* in that the extensional type  $(e \rightarrow (e \rightarrow t))$  both transitive verbs are typically assigned is too simplistic. Montague was implicitly suggesting that a much more sophisticated ontology (i.e., a more complex type system) is needed, one that would in fact yield different types for *find* and *paint*. One reasonable suggestion for the types of *find* and *paint*, for example, could be as follows:

- (18)  $\text{find} :: (e_{\text{Animal}} \rightarrow (e_{\text{Thing}} \rightarrow t))$   
 (19)  $\text{paint} :: (e_{\text{Human}} \rightarrow (e_{\text{Representation}} \rightarrow t))$

Thus instead of the flat type structure implied by  $(e \rightarrow (e \rightarrow t))$ , the types of *find* and *paint* should reflect our commonsense belief that we can always speak of some *Animal* that found something (i.e., any *Thing* whatsoever), and of a *Human* that painted some illustration, or as we called it here a *Representation*. Before we proceed, however, we point out that throughout, we will use this font for concept types in the ontology, and this font for predicate names. Thus,  $x :: \text{LivingThing}$  means  $x$  is an object/entity of type *LivingThing* and  $\text{apple}(x)$  means the predicate or property *apple* is true of  $x$ . Note, further, that in a flat-type system, the expression  $(\exists x)(\text{unicorn}(x) \wedge \text{found}(j,x))$  is equivalent to the typed expression  $(\exists x :: \text{Entity})(\text{unicorn}(x) \wedge \text{found}(j :: \text{Entity}, x))$  since in flat type system there is only one type of entity. With this background, the correct translations of (18) and (19) and the corresponding inferences can now be given as follows:

- (20)  $(\exists x :: \text{Thing})(\text{unicorn}(x) \wedge \text{found}(j :: \text{Rational}, x))$   
 $\Rightarrow (\exists x :: \text{Thing})(\text{unicorn}(x))$   
 $\Rightarrow (\exists x :: \text{Thing})(\text{found}(j :: \text{Rational}, x))$

- (21)  $(\exists x: \text{Representation})(\text{unicorn}(x) \wedge \text{painted}(j: \text{Rational}, x))$   
 $\Rightarrow (\exists x: \text{Representation})(\text{unicorn}(x))$   
 $\Rightarrow (\exists x: \text{Representation})(\text{painted}(j: \text{Rational}, x))$

Adding a rich type structure to the semantics, it seems, provides a reasonable solution to Quine's puzzle, as the correct inferences can now be made: if John found a unicorn, then one can indeed infer that an actual unicorn exists<sup>2</sup>. However, the painting of a unicorn only implies the existence of a representation (an illustration) of something we call a unicorn! Stated yet in other words, (7) implies that a unicorn *Thing* (including perhaps a unicorn *Toy*) exists, while (8) implies a unicorn *Representation* exists. There are two points that this discussion intends to emphasize: (i) is the need for a rich type structure to solve a number of problems in the semantics of natural language; and (ii) that this type structure is actually systematically discovered by an analysis of how ordinary language is used to talk about the world.

## 5 Language, Logic, Ontology and Commonsense

Our work here has been motivated by the (rather strong) claim of Richard Montague (see the paper on ELF in (Thomasson, 1974)) that there is no theoretical difference between formal and natural languages. If does turn out that Montague is correct (as we believe to be the case), then there should exist a formal system, much like arithmetic, or any other algebra, for concepts, as has been advocated by a number of authors, such as Cresswell (1973) and Barwise (1989), among others. What we are arguing for here is a formal system that explains how concepts of various types combine, forming more complex concepts in a formal, strongly-typed system. To illustrate, consider the following:

- (22)  $\text{artificial} :: \text{NaturalKind} \rightarrow \text{Artifact}$   
 (23)  $\text{flower} :: \text{Plant}$   
 (24)  $\text{flower} :: \text{Plant} \supset \text{LivingThing}$   
 (25)  $\text{flower} :: \text{Plant} \supset \text{LivingThing} \supset \text{NaturalKind}$   
 (26)  $\text{artificial flower} :: \text{Artifact}$

What the above says is the following: *artificial* is a function that takes a *NaturalKind* and returns an *Artifact* (22); a *flower* is a *Plant* (23); a *flower* is a *Plant* which in turn is a *LivingThing* (24); a *flower* is a *Plant*, which is a *LivingThing*, which in turn is a *NaturalKind* (25); and, finally, an *artificial flower* is an *Artifact* (26). Therefore, 'artificial c', for some *NaturalKind* c, should in the final analysis have the same properties that any other *Artifact* has. Thus, while a *flower*, which is of type *Plant*, and is therefore a *LivingThing*, grows, lives and dies like any other *LivingThing*, an

<sup>2</sup> Of course, in such a type system we would have  $\text{Rational} \supset \text{Animal}$  and therefore John, an entity of type *Rational*, can be the subject of *found* which expects an entity of type *Animal*.

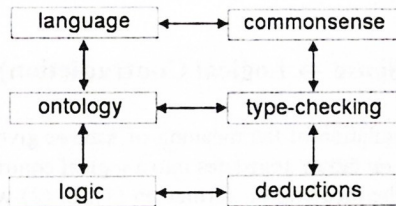


*artificial flower*, and like any other *Artifact*, is something that is manufactured, does not grow, does not die, but can be assembled, destroyed, etc. The concept algebra we have in mind should also systematically explain the interplay between what is considered commonsense at the linguistic level, type checking at the ontological level, and deduction at the logical level. For example, the concept *artificial car*, which is a meaningless concept from the standpoint of commonsense, is ill-typed since *Car* is an *Artifact*, and *Artifact* does not unify with *NaturalKind* – neither type is a sub-type of the other.

The concept *former father*, on the other hand, which is also a meaningless concept from the standpoint of commonsense, escapes type-checking since *father*, which is a *Role*, is a type that *former* expects as shown in (29) below.

(29) *former* :: *Role* → *Role*

However, although *former father* escapes type-checking, the fact that this a meaningless concept from the standpoint of commonsense, is ultimately detected at the logical level by resulting in a contradiction as shown in the appendix. Thus what is meaningless at the linguistic level should be flagged at the type-checking level, or, if happens to escapes type-checking, such as *former father*, it should eventually result in a logical contradiction at the logical level (see the appendix concerning *former father*). The picture we have in mind can therefore be summarized as shown in the figure below.



## 6 Concluding Remarks

A number of problems in the semantics of natural language can be resolved in a compositional semantic framework if a rich type system that models an ontology of commonsense concepts can be assumed. If this were to happen, it would mean that there is a formal system that underlies natural language and that a concept algebra must exist. This subsequently means that the ontology we have in mind must be systematically discovered and cannot be invented, as has been argued by Saba (2001). In this paper we have shown that assuming such a rich type systems can help resolving a number of challenging problems in the semantics of natural language. For lack of space, in this paper we could not discuss the nature of this ontological structure, the corresponding strongly-typed meaning algebra, and how this structure might be discovered rather than invented, using natural language itself as a guide in this process. Some of these issues are discussed in some detail in Saba (2006).

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## Appendix: (Non-Sense $\rightarrow$ Logical Contradiction)

Using the logical formulation of the meaning of *former* given above, we show here how the concept 'former father' translates into a logical contradiction.

First, we reiterate the meaning of 'former' in (1). In (2) we state the fact that the role type *Father* has an essential temporal property, namely that once someone is a father they are always a father. The deductions that follow should be obvious.

1.  $(\forall x : \text{Role})(\text{former}(x) \equiv_{df} \lambda P[(\exists t)((t < \text{now}) \wedge P(x, t) \wedge \neg P(x, \text{now}))])$
2.  $(\forall x)((\exists t_1)(\text{father}(x, t_1) \supset (\forall t_2)((t_2 > t_1) \supset \text{father}(x, t_2))))$
3.  $(\exists t)((t < \text{now}) \wedge \text{father}(x, t) \wedge \neg \text{father}(x, \text{now}))$  (1) applied on *father*
4.  $(t < \text{now}) \wedge \text{father}(x, t) \wedge \neg \text{father}(x, \text{now})$  EI of (3)
5.  $\text{father}(x, t)$   $\wedge$  - elimination of (4)
6.  $(\exists t_1)(\text{father}(x, t_1) \supset (\forall t_2)((t_2 > t_1) \supset \text{father}(x, t_2)))$  UG of (2)
7.  $\text{father}(x, u) \supset (\forall t_2)((t_2 \geq u) \supset \text{father}(x, t_2))$  EI of (6)
8.  $(\forall t_2)((t_2 > t) \supset \text{father}(x, t_2))$  (5), (7) and MP
9.  $(t_2 > t) \supset \text{father}(x, t_2)$  UG of (8)
10.  $(t < \text{now})$   $\wedge$  - elimination of (4)
11.  $\text{father}(x, \text{now})$  (9), (10) and MP
12.  $\neg \text{father}(x, \text{now})$   $\wedge$  - elimination of (4)
13.  $\perp$  (11) and (12)